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Why have we Never Observed the Massless Charged Particle?

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In this paper we try to explore the possible contact between quantum gravity and the least mass of a charged particle in de Sitter spacetime. The effect of Generalized Uncertainty Principle (GUP) on the thermodynamics of de Sitter spacetime is discussed in a heuristic manner. We find a maximal entropy/probability that corresponds to the absence of charge of a massless particle. Furthermore, the holographic principle provides a possible lower limit to the mass of a charged particle.

KEY WORDS: electronic charge; generalized uncertainty principle; cosmological constant; holographic principle.

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1. INTRODUCTION

It is well known that the electron is the lightest charged particle in nature. However, to our best knowledge, the possibility of the charged particle lighter than the electron is not excluded by a fundamental principle, although it has never been observed. On the contrary, the massless charged particle appears in the supersymmetric theory with an unbroken sector of Abelian gauge symmetry (Kazinski and Sharapov, 2003). We cannot help asking: is this a real story? is there a lower limit to the mass of a charged particle? what is its value? is it arbitrarily small? The answers appear to depend on the developments of the particle physics and quantum field theory. On the other hand, these problems may be related to the quantum gravity. This is because more and more evidences indicate that a real world should be described by a theory of quantum gravity with finite number of degrees of freedom. For example, cosmological constant or vacuum energy problem, as a bridge across quantum field theory and general relativity, is related to holographic principle (Cohen *et al.*, 1999; Horava and Minic, 2000; Thomas, 2002). Usual particle physics, based on field theory with infinitely many degrees of freedom, should be corrected in a certain extent.

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In this paper, we attempt to gain a primary knowledge about the possible contact between these problems and quantum gravity in de Sitter spacetime. It involves two aspects: generalized uncertainty principle (GUP) and the holographic principle. We will perform the discussions in the de Sitter spacetime, since the accelerating expansion of current universe implies a positive cosmological constant (as one of the candidates for dark energy). A more important reason is that the horizon of de Sitter spacetime provides a natural bound on the number of degrees of freedom. The paper is organized as follows. In Section 2, we consider the GUP corrected thermodynamics of de Sitter spacetime disturbed by a charged particle. We observe that vanishing charge corresponds to the greatest probability when the particle mass is zero, In Section 3, we argue that the least mass of a charged particle is determined by the N bound of the holographic principle in the de Sitter spacetime.

2. GUP CORRECTED THERMODYNAMICS OF A DE SITTER SPACETIME

Astronomical observations show that the current universe is dominated by the dark energy which drives the cosmic acceleration. As one of the candidates for the dark energy, the cosmological constant is the simplest choice. It is reasonable to regard the de Sitter spacetime as the background we consider. We start with a Reissner-Nordström-de Sitter spacetime

$$ds^2 = -f(r, m, Q, \Lambda)dt^2 + f^{-1}dr^2 + r^2 d\Omega, \tag{1}$$

where $f(r, m, Q, \Lambda) = 1 - 2m/r + Q^2/r^2 - \Lambda r^2/3$, Λ, m, Q are the cosmological constant, the mass and the electronic charge, respectively. Here we consider a charged particle rather than a black hole, so there is only a cosmological horizon located by r_c, which satisfies

$$1 - \frac{2m}{r_c} + \frac{Q^2}{r_c^2} - \frac{1}{3}\Lambda r_c^2 = 0. \tag{2}$$

The surface gravity is given by

$$\begin{aligned} \kappa &= -\frac{1}{2} f'(r_c) \\ &= -\frac{m}{r_c^2} + \frac{Q^2}{r_c^3} + \frac{\Lambda}{3} r_c. \end{aligned} \tag{3}$$

It is well known that there is no massless charged particle in nature. This is an observed fact. Equation (1) will be an un-physical ghost if the mass is exactly zero, while $Q \neq 0$. However, we hope it can be excluded by a fundamental principle. In other words, the absence of massless charged particle is not only regarded as an observed fact but also expected to be enforced by a fundamental principle. Concretely, we want to know whether the GUP can tell us something about (1), if the charge Q is not set to be zero in advance but treated as a variable. We wait for GUP to single out the right one from other unphysical possibilities.

GUP is the generalization of Heisenberg's uncertainty principle. According to the usual quantum mechanics, the minimal position uncertainty yields $\Delta x \sim 1/\Delta p$. a particle can be located with arbitrary, accuracy, if Δp is large enough. However, it is a common belief that there is a fundamental scale in quantum gravity (Planck length $l_{\mathrm{p}} = \sqrt{(G\hbar/c^3)}$, which is the minimal length we can probe in principle). In order to match up the quantum gravity, Heisenberg's principle is likely to suffer some modifications (Adler *et al.*, 2001; Ahluwalia, 2000; Garay, 1995; Gross and Mende, 1986; Kempf *et al.*, 1995; Li and Shen, 2004; Veneziano, 1986), at least on a phenomenological level. The simplest generalization is

$$\Delta x \Delta p \geq 1 + \lambda \pi^2 (\Delta p)^2, \tag{4}$$

where λ is a parameter, which is supposed to be proportional to the Planck area. The first term on the r.h.s. of (4) is just the contribution of Heisenberg's uncertainty principle. The nontrivial feature of the GUP comes from the second term, which implies a new duality of momentum and position. Since $\Delta x \sim \Delta p$, it agrees with the feature of a string: the energy increases with its length. In fact, such a generalization of uncertainty principle was first derived from the string theory (Gross and Mende, 1986; Veneziano, 1986).

It has been argued that the GUP influences the Hawking radiation (Adler *et al.*, 2001). A possible formula for the modified temperature of black hole is given by (Li and Shen, 2004)

$$T = \frac{2}{\beta + \sqrt{\beta^2 - 4\lambda\pi^2}}, \tag{5}$$

where $\beta = 2\pi\kappa^{-1}$, κ is the surface gravity at the event horizon. This formula is different from the argument of Adler *et al.* (2001). It is not only valid to the Schwarzschild black hole, but also applicable to a general class of black holes as well as the cosmological horizon. Corresponding to the modification to temperature, the corrected entropy is given by (Li and Shen, 2004)

$$S = \frac{1}{8} \int F(\kappa) dA, \tag{6}$$

where

$$F(k) = 1 + \sqrt{1 - \lambda\kappa^2}. \tag{7}$$

As presented by (3), κ is a function of four variables (m, Q, Λ, r_c). When we set $m = 0$, the entropy formula (6) is expressed by

$$S = \frac{1}{8}\int F(Q, \Lambda, r_c)dA, \tag{8}$$

To prepare for the further discussion, we directly provide the following relations

$$r_c = \sqrt{\frac{3}{\Lambda}}\sqrt{\frac{1+\sqrt{1+4\Lambda Q^2}}{2}}, \tag{9}$$

$$\kappa = \frac{Q^2}{r_c^3} + \frac{\Lambda r_c}{3}, \tag{10}$$

$$B \equiv \frac{Q}{r_c\sqrt{(1+4\Lambda Q^2/3)}}, \tag{11}$$

$$\frac{\partial\kappa}{\partial Q} = \left(\frac{2\Lambda}{3} + \frac{1}{r_c^2}\right)B, \tag{12}$$

$$\frac{\partial^2\kappa}{\partial Q^2} = r_c^{-3}\big(2\Lambda r_c^2+3\big)\big(2\Lambda r_c^2-3\big)^{-1}\big[1+\big(1-2\Lambda r_c^2\big)B^2\big] - 2r_c^{-3}B^2, \tag{13}$$

where $m = 0$. As an observed fact, Q should be zero when the mass vanishes. However, we expect to find the more fundamental reason hidden behind this fact. Therefore Q is regarded as a variable and the entropy (8) becomes a functional of Q. The most probable Q is one that the de Sitter spacetime possesses the maximum entropy (corresponding to the greatest probability), which satisfies

$$\frac{\delta S}{\delta Q} = 0, \quad \frac{\delta^2 S}{\delta Q^2} < 0, \tag{14}$$

which actually requires

$$\frac{\partial F}{\partial Q} = \frac{\partial F}{\partial\kappa} \times \frac{\partial\kappa}{\partial Q} = 0, \quad \frac{\partial^2 F}{\partial Q^2} < 0. \tag{15}$$

However,

$$\frac{\partial F}{\partial\kappa} = -\frac{\lambda\kappa}{\sqrt{1-\lambda\kappa^2}} \neq 0, \tag{16}$$

so

$$\frac{\partial\kappa}{\partial Q} = 0 \Leftrightarrow Q = 0. \tag{17}$$

It is just what we expect. The vanishing charge is indeed prior to other possibilities, for a massless particle. However, our observation is not a trivial repeat of the observed fact, because

$$\frac{\partial^2 F}{\partial Q^2} = \frac{\partial F}{\partial \kappa} \times \frac{\partial^2 \kappa}{\partial Q^2} = -\frac{\lambda \Lambda^2}{\sqrt{9 - 3\lambda\Lambda}} < 0 \tag{18}$$

exhibits that S is a maximum. It implies that the absence of charge of a massless particle is related to the quantum gravity effects. A nonzero parameter λ and the minus sign of Eq. (7) are crucial to our observation. We shall not confirm that $Q = 0$ corresponds to the greatest probability, if $\lambda = 0$ or it does not appear in Eq. (7). We shall not find the probable value of Q if the quantum gravity provides a positive correction to entropy [i.e. $-\lambda\kappa^2$ is replaced by $+\lambda\kappa^2$ in Eq. (7)] and then S is a minimum.

It is interesting that the second law of thermodynamics (corrected by quantum gravity) "knows" the massless particle has no charge! Since this is related to the de Sitter background, it is reasonable to hypothesize that there is a possible connection between the cosmological constant and the least mass of a charged particle. A quantitative relation will be discussed in the following.

3. HOLOGRAPHIC PRINCIPLE CONSTRAINS THE LEAST MASS OF A CHARGED PARTICLE

It is generally believed that the holographic principle is an important aspect of quantum gravity (Susskind, 1995; 't Hooft, 1993), although a perfect theory has not been established. The original version of holographic principle states that the entropy of a system is bounded by the area of its boundary, $S \leq A/4$. However, this statement should be improved in the presence of a positive cosmological constant. This is because the cosmological horizon has the entropy proportional to its area. The horizon area decreases with the increasing matter in an asymptotic de Sitter spacetime. Obviously, the entropy bound represented by the horizon area can easily be violated by the systems that consist of the cosmological horizon and matter. An improved version named N bound (Banks, 2000; Bousso, 2000) states that a universe with a positive cosmological constant is described by a quantum theory of gravity with the finite number of degrees of freedom $N = 3\pi/\Lambda$, and its entropy is not greater than $S = N$. It is just the entropy of a pure de Sitter spacetime. Since the horizon area can be identified with entropy, N bound means that the size of the world with $\Lambda (> 0)$ cannot exceed the pure de Sitter spacetime. It can easily be accepted, since the existence of neutral matter always makes the cosmological horizon shrink. On the contrary, the effect of electronic charge increases the cosmological horizon.

Here we consider the thermodynamics of an asymptotic de Sitter spacetime as a constraint on the electron mass, declining those that could lead to a violation of the N bound. We consider a charged particle with mass m and charge e in the de Sitter spacetime, which is still described by the Reissner-Nordström-de Sitter (RNdS) metric presented by (1). Here the charged body we consider is not a black hole, so there is only a cosmological horizon located by $f(r_c) = 0$, which has four complicated solutions and one of them corresponds to the cosmological horizon. However, we need not know the exact expression of this solution. It is sufficient for our aim to know Eq. (2). The surface gravity is given by (3). A direct calculation reveals the following equation

$$dm = -\frac{\kappa_c}{8\pi} dA_c + \frac{ede}{r_c} - \frac{r_c^3}{6} d\Lambda, \tag{19}$$

where $A' = 4\pi r_c^2$ is the area of the cosmological horizon. The entropy reads $S = A/4$. The above equation is similar to the first law of thermodynamics of black holes, except that there is a minus sign of the first term on the right hand side. The last term is the contribution from the change in vacuum energy density (or cosmological constant). From Eq. (19) we obtain

$$\left(\frac{\partial S}{\partial m}\right)_{\Lambda,e} = -\frac{2\pi}{\kappa_c}, \tag{20}$$

which means that the entropy increases with the decreasing mass, for any given Λ and e. However, according to N bound, the entropy of the universe with a positive cosmological constant doesn't exceed the entropy of a pure de Sitter spacetime, $S_{\max} = 3\pi/\Lambda$. This demands that the radius of the cosmological horizon satisfy $r_c \leq \sqrt{3/\Lambda}$. Substituting the upper bound into (2), we obtain a relation between the cosmological constant and the least mass of a charged particle,

$$2m = e^2\sqrt{\Lambda/3}. \tag{21}$$

We can understand (21) in such a manner: the entropy, measured by the area of cosmological horizon, increases with the decreasing mass. For a given e, there must exist a minimal mass to maintain the N bound. This bound would be violated if the mass of a charged particle is arbitrarily small. To see this, we substitute the ansatz $r_c = r_0(1+\delta) = \sqrt{3/\Lambda}(1+\delta)$ into (2), and obtain

$$r_0^2\delta^4 + 4r_0^2\delta^3 + 5r_0^2\delta^2 + 2r_0(r_0+m)\delta + 2mr_0 - e^2 = 0, \tag{22}$$

where the last two terms deserves attention on the left hand side of the above equation. There must exist a positive root, if $2mr_0 < e^2$. The entropy is measured by $S = \pi r_c^2 > 3\pi/\Lambda$, and then violates the N bound. The critical point $2mr_0 = e^2$ is just (21).

4. SUMMARY AND DISCUSSION

In summary, the possible effects on the least mass of a charged particle is discussed from two aspects of quantum gravity: GUP and the holographic principle. GUP corrects the thermodynamics of a disturbed de Sitter spacetime (without black hole), where the "charge" of a massless particle is not set to be zero but treated as a variable Q. the correction to the entropy of horizon is obtained, and the probability of absence of charged massless particle is indeed prior. This implies that the real mass of charged particles cannot be arbitrarily small. The holographic principle further associates the least mass of a charged particle with the cosmological constant and the fine structure constant.

The following is devoted to some remarks on present work.

(a) Equation (21) is too naive to impose a realistic constraint on the mass of a charged particle, because the mass scale given by (21) is much less than the observed electron mass. However, we can improve on (21) when the number and species of charged particles are taken into account. For example, considering N charged particles, (21) can be changed to $2mr_0 = Ne^2$. The lower limit to the mass of a charged particle is elevated.

(b) The effects of the GUP and the holographic principle are discussed separately in this paper. The relation between GUP and the holographic principle is an open problem, which has not been involved in preceding discussions. It is noted that GUP suppresses the number of quantum states (Chang *et al.*, 2002; Rama *et al.*, 2001), and removes the divergent quantum correction to the black hole entropy (Li, 2002). This somewhat accords with the fundamental feature of holographic principle: a theory with finite number of degrees of freedom. N bound may be corrected if the effect of GUP is considered. However, the correction to the lower limit (21) should be inessential.

(c) The present work is mainly based on some theoretical considerations of quantum gravity. Although the origin of electronic mass is still not involved in this paper, we try to express such an idea: the electronic mass is perhaps related to the quantum gravity. This has been displayed by the preceding discussions, especially the N bound constrains the least mass of a charged particle. Quantum fields are commonly treated as the systems with infinite number of degrees of freedom. However, the holographic principle from quantum gravity means that the real world should be described by a theory with finite degrees of freedom. Therefore, it is reasonable that holographic principle and quantum gravity influence the usual quantum field theory and particle physics.

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REFERENCES

Adler, R. J., Chen, P. S., and Santiago, D. I. (2001). The generalized uncertainty principle and black hole remnants. *General Relativity and Gravity* **33**, 2101.

Ahluwalia, D. V. (2000). Wave-Particle duality at the Planck scale: Freezing of neutrino oscillations. *Physics Letters A* **275**, 31.

Banks, T. (2000). Cosmological Breaking of Supersymmetry? hep-th/0007146.

Bousso, R. (2000). Positive vacuum energy and the *N*-bound. *Journal of High Energy Physics* **0011**, 038.

Chang, L. N., Minic, D., Okmura, N., and Takeuchi, T. (2002). The effect of the minimal length uncertainty relation on the density of states and the cosmological constant problem. *Physical Review D* **65**, 125028.

Cohen, A. G., Kaplan, D. B., and Nelson, A. E. (1999). Effective field theory, black Holes, and the cosmological constant. *Physical Review Letter* **82**, 4971.

Garay, L. J. (1995). Quantum gravity and minimum length. *International Journal of Modern Physics A* **10**, 145.

Gross, D. J. and Mende, P. F. (1988). String theory beyond the Planck scale. *Nuclear Physics B* **303**, 407.

Horava, P. and Minic, D. (2000). Probable values of the cosmological constant in a holographic theory. *Physical Review Letter* **85**, 1610.

Kazinski, P. O. and Sharapov, A. A.(2003). Radiation reaction for a massless charged particle. *Classical and Quantum Gravity* **20**, 2715.

Kempf, A., Mangano, G., and Mann, R. B. (1995). Hilbert space representation of the minimal length uncertainty relation. *Physical Review D* **52**, 1108.

Li, X. (2002). Black hole entropy without brick walls. *Physics Letters B* **540**, 9.

Li, X. and Shen, Y. G. (2004). Probable ratio of the vacuum energy in a Schwarzchild-de Sitter space. *Physics Letters A* **324**, 255.

Rama, S. K. (2001). Some consequences of the generalised uncertainty principle: Statistical mechanical, cosmological, and varying speed of light. *Physics Letters B* **519**, 103.

Susskind, L. (1995). The world as a hologram. *Journal of Mathmatical Physcis* **36**, 6377.

Thomas, S. (2002). Holographic vacuum energy. *Physical Review Letter* **89**, 081301.

’t Hooft, G. (1993). Dimensional reduction in quantum gravity. gr-qc/9310026.

Veneziano, G. (1986). A string nature needs just two constants. *Europhysic Letter* **2**, 199.